



SECTION II: KINETICS AND BIOREACTOR DESIGN:
LESSON 9.2. - Enzymatic kinetics, microbial kinetics and metabolic stoichiometry – Alive cells in bioprocesses



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AIMS FOR TODAY'S LESSON

1.- ABOUT GROWTH

How to express (microbial) growth.

2.- ABOUT MICROORGANISMS in BATCH PROCESSES:

Steps along a batch growth - Balanced growth?

Yields - Kinds of products - Oxygen necessities

3.- ABOUT MICROORGANISMS in CONTINUOUS PROCESSES:

Perfect mixing .

Chemostat / Turbidostat.

1. PROCESSES via CELLS

ALIVE CELLS IN BIOPROCESSES

STOICHIOMETRY

Many enzymatic reactions: Metabolism
 Complex scheme of reactions: **need simplification**
 ANALYSIS: stoichiometric study

Substrates $\xrightarrow{\text{Cells}}$ CELLS

Substrates $\xrightarrow{\text{Cells}}$ Products

Substrates $\xrightarrow{\text{Cells}}$ Energy

KINETIC MODELS

Each **KEY COMPUND** for each reaction
Autocatalytic reactions
 Slow process → higher reactor volume or reaction time
 Depending on cell type: chemo-, photo-, heterotroph, autotroph
 O₂ (aerobic, anaerobic), T, pH
 cell state: phase growth, viability, stability (GMO)
Empirical equations → Problems in Scaling up
NEED OF SIMPLIFICATION: Structure, segregation

Simplified reaction scheme

Many reaction rates, kinetic parameters (macroscopic)

Empirical kinetic model: key components

1. PROCESSES via MICROORGANISMS

Changes in physico-chemical environment result in **different responses in microorganism growth**.

The proper medium allows organisms to extract necessary nutrients in order to cover different metabolic necessities:

- **Energy requirements**
- **Biosynthesis**
 - Product generation.
 - Biomass rise.

1.- GROWTH

2.- CELLS IN BATCH PROCESSES

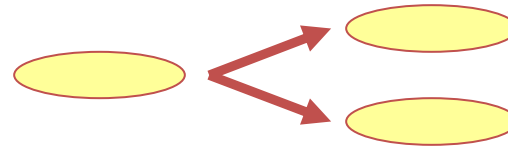
3.- CELLS IN CONTINUOUS PROCESSES

1.- GROWTH

2. MICROBIAL GROWTH

GROWTH HYPOTHESIS?:

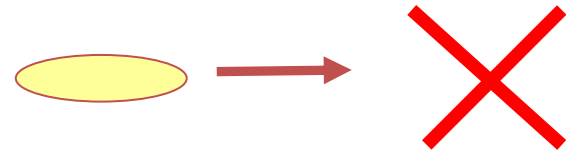
- Each cell leads to two cells.



- Cells unable to reproduce are not taken into account.

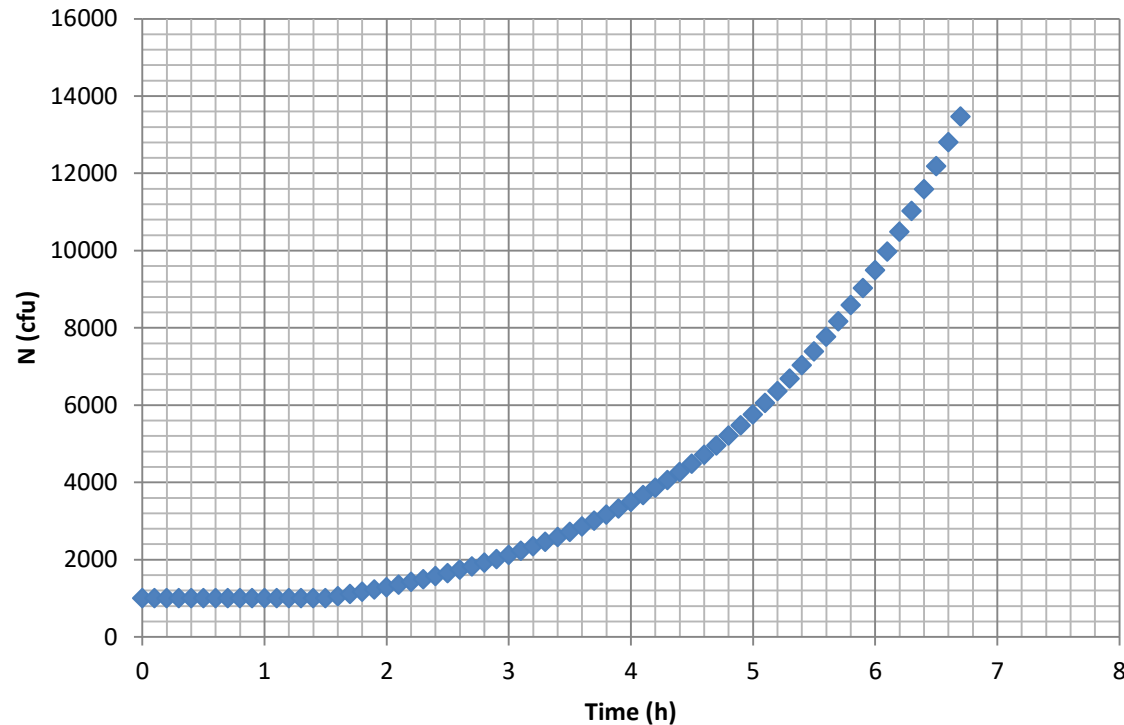


- Cells dying are not taken into account



2. MICROBIAL GROWTH

GROWTH HYPOTHESIS?:



2. MICROBIAL GROWTH

Substrates + Cells → Extracellular products + New Cells



It is an autocatalytic process

Rate of growth is directly proportional to cell concentration; in other words:

$$r = \frac{dX}{dt} = k \cdot X$$

2. MICROBIAL GROWTH

$$r = \frac{dX}{dt} = k \cdot X$$

First – order kinetics

Constant of proportionality **specific growth rate**:

$$r = \frac{dX}{dt} = \mu_{net} \cdot X; \quad \mu_{net} = \frac{1}{X} \cdot \frac{dX}{dt}$$
$$\mu_{net} = \mu_g - \mu_d$$

2. MICROBIAL GROWTH

$$r = \frac{dX}{dt} = \mu_{net} \cdot X; \quad \mu_{net} = \mu_g - \mu_d$$

- **X**: biomass concentration (g/L).
- **t**: time (h).
- μ_{net} : specific net rate (h^{-1}).

Difference between:

μ_g : specific growth rate (h^{-1}).

μ_d : specific cell death rate (or endogenous metabolism) (h^{-1}).

2. MICROBIAL GROWTH

In a different way:

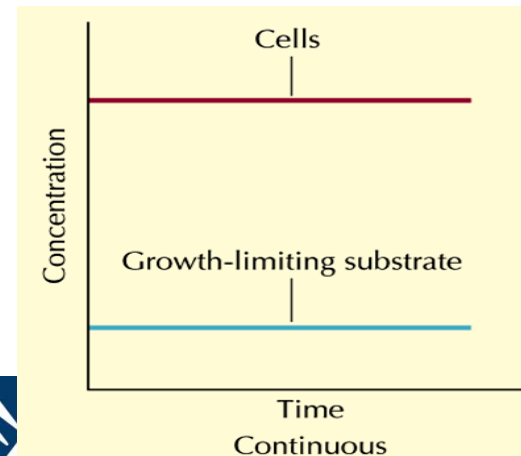
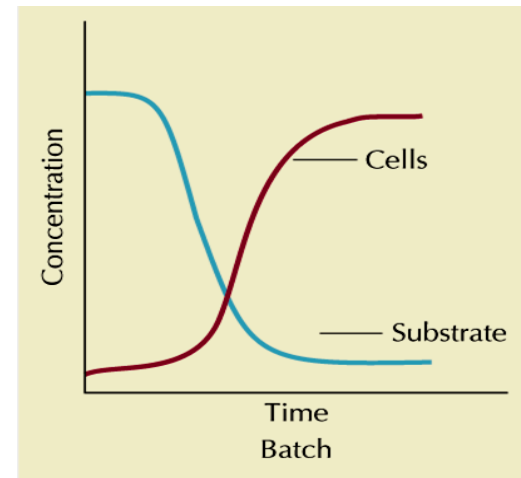
$$\mu_R = \frac{1}{N} \cdot \frac{dN}{dt}; \quad \mu_R = \mu_g - \mu_d$$

- **N**: cell concentration (cfu/L; spores/L).
- **t**: time (h).
- μ_R : specific net replication (or duplication) rate(h^{-1}).

2. MICROBIAL GROWTH – Two possibilities

Equations describing growth kinetics change depending on:

- we are using a **batch process**.
- we are using a **continuous process**.



1.- GROWTH

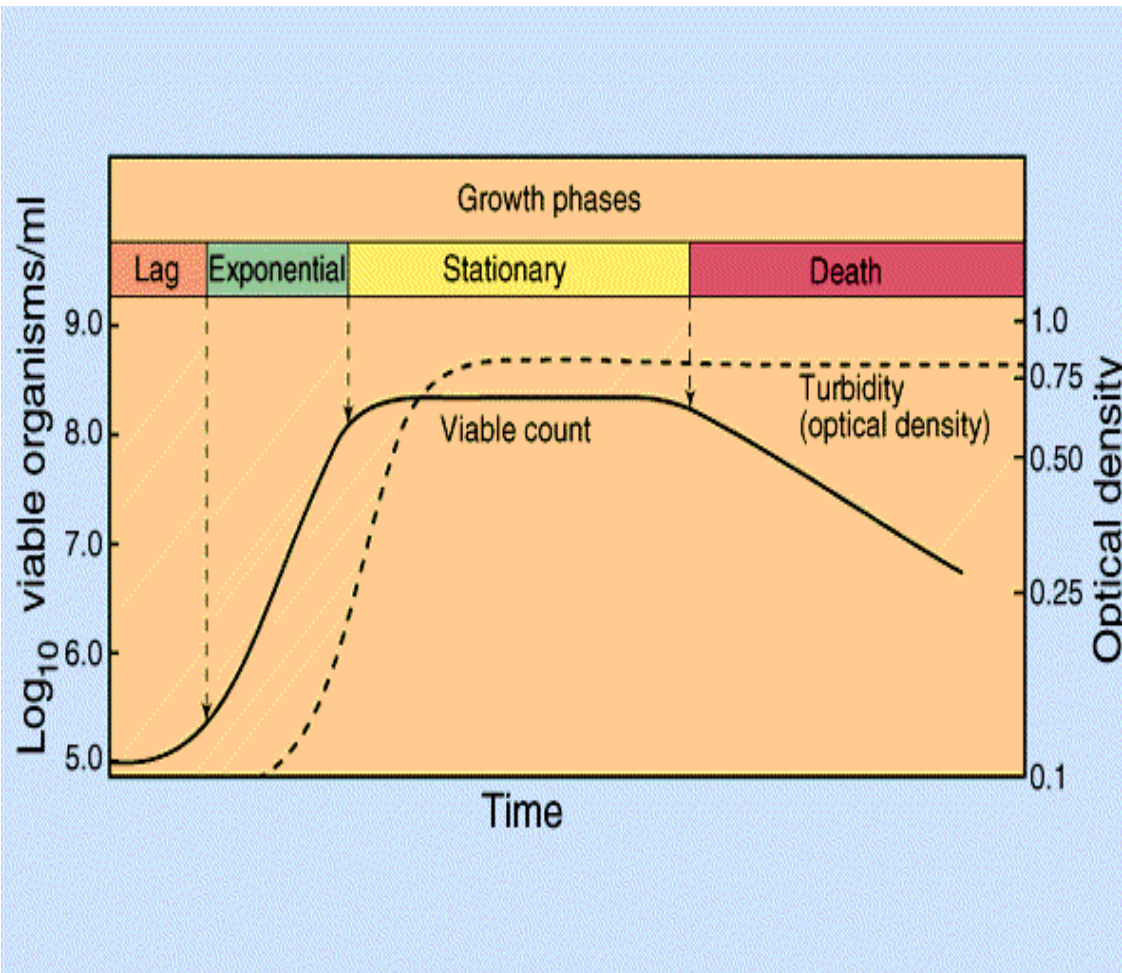
2.- CELLS IN BATCH PROCESSES

3.- CELLS IN CONTINUOUS PROCESSES

A photograph of an industrial facility featuring several large, cylindrical, light-colored storage tanks. The tanks are arranged in a row, and a metal staircase is visible on the right side of the foreground tank. In the background, there are other industrial buildings and a clear blue sky with scattered white clouds. A dark blue horizontal bar is overlaid on the image, containing the section header text.

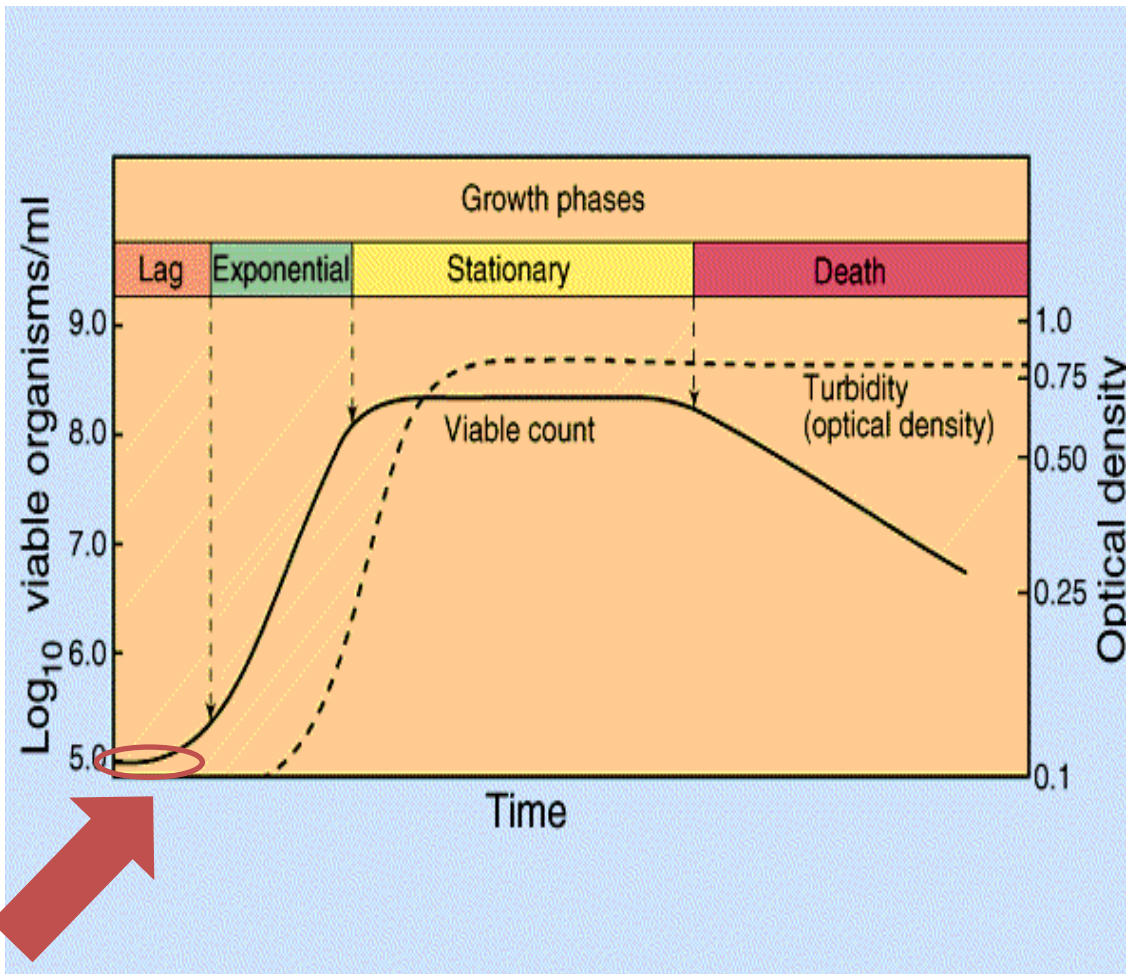
2.- CELLS IN BATCH PROCESSES

3. MICROBIAL BATCH PROCESS



Phase	
I	Lag (delay or latency)
II	Acceleración
III	Exponential (logaríthmic)
IV	Deceleration
V	Stationary
VI	Decay or death

3. MICROBIAL BATCH PROCESS



Phase

I Lag (delay or latency)

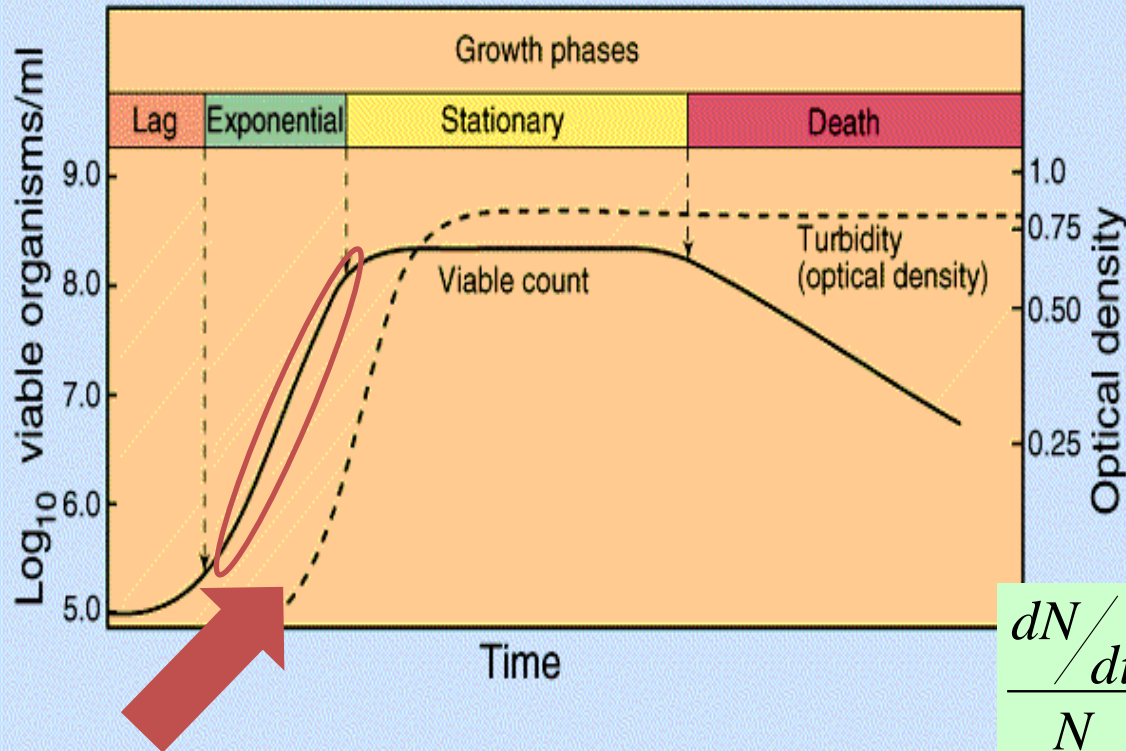
DURATION:

- Similarity between previous and current medium.
- Type of microorganism.
- This phase always exists.

DIAUXIC GROWTH:

- More than one source of C.
- Metabolic adaptations
- “Multiple phases of latency”

3. MICROBIAL BATCH PROCESS



Phase	
III	Exponential (logarithmic)

Balanced growth

- ONLY WITHIN THIS PHASE

$$\frac{dN/dt}{N} = \frac{dDNA/dt}{DNA} = \frac{dPROT/dt}{PROT} = \mu$$

3. MICROBIAL BATCH PROCESS

Phase	
III	Exponential (logarithmic)

Balanced growth

$$r = \frac{dX}{dt} = \mu_{net} \cdot X; \quad \mu_{net} = \frac{1}{X} \cdot \frac{dX}{dt} \Rightarrow X = X_0 \cdot e^{\mu_{net} \cdot t}$$

$$\mu_{net} = \mu_g - \mu_d$$

$$t_g = \frac{\ln(2)}{\mu_{net}} \text{ or } t'_g = \frac{\ln(2)}{\mu_R}$$

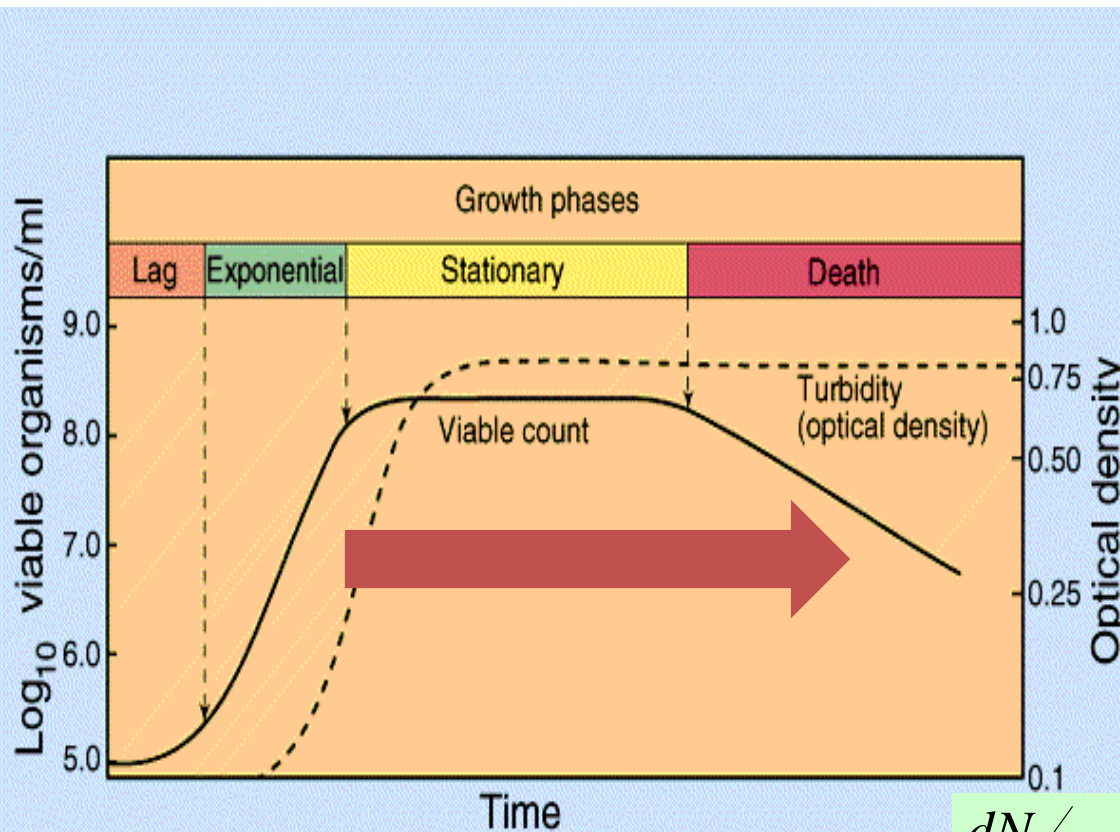
$$\mu_R = \frac{1}{N} \cdot \frac{dN}{dt}$$

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3. MICROBIAL BATCH PROCESS



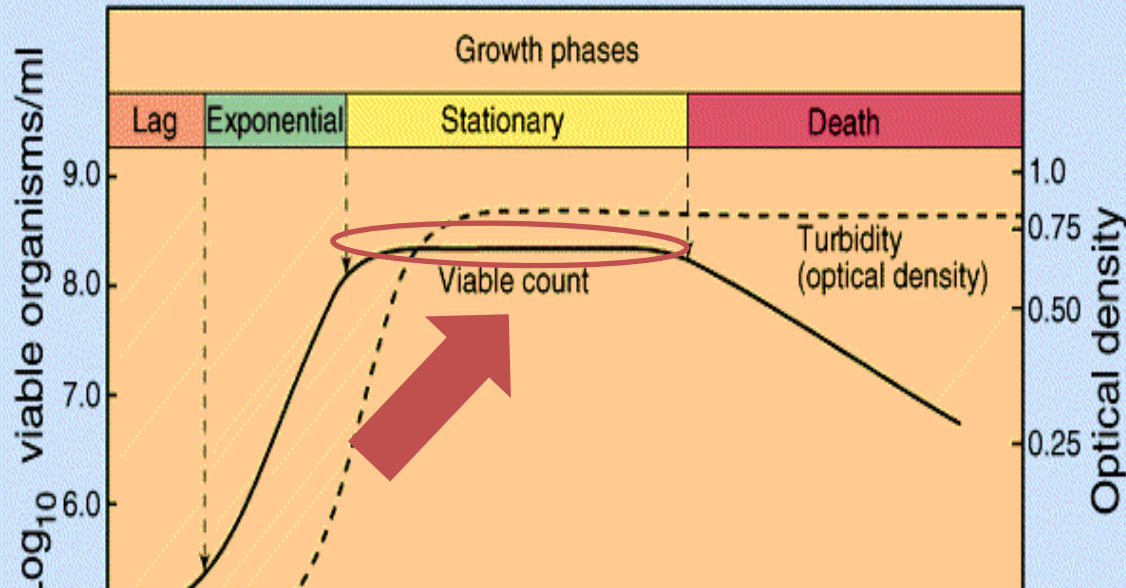
Phase	
IV	Deceleration
V	Stationary
VI	Decay or death

STRESS

$$t_g = \frac{\ln(2)}{\mu_{net}} \neq t'_g = \frac{\ln(2)}{\mu_R}$$

$$\frac{dN/dt}{N} \neq \frac{dDNA/dt}{DNA} \neq \frac{dPROT/dt}{PROT} \neq \mu$$

3. MICROBIAL BATCH PROCESS



$$\left. \frac{dX}{dt} \right|_{\text{endogenous metabolism}}$$

$$= -k_d \cdot X \Leftrightarrow X = X_{S0} \cdot e^{-\mu_d \cdot t}$$

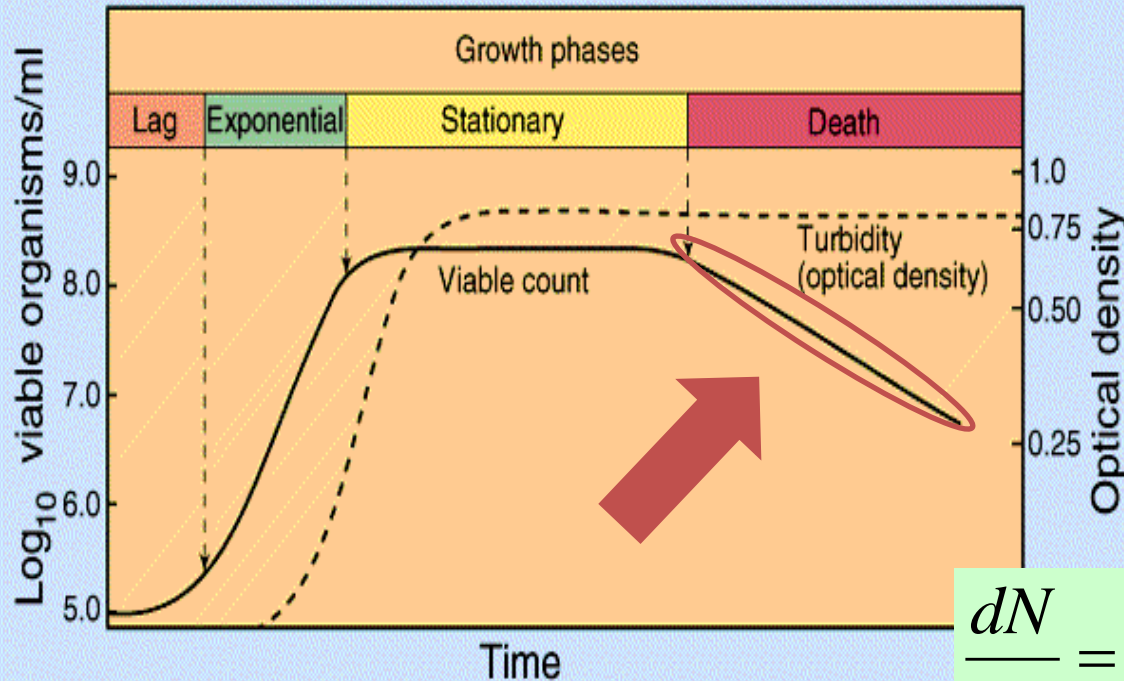
Phase	
V	Stationary

ENDOGENOUS METABOLISM

Biomass conversion into energy:

- X_{S0} : initial biomass concentration at the beginning of stationary phase (g/L).
- t : time (h).
- μ_d : specific rate for endogenous metabolism (h^{-1}).

3. MICROBIAL BATCH PROCESS



Phase	
VI	Decay or death

- Absolut dependence of endogenous metabolism.
- Cell death and lysis → number of viable cells falls exponentially.

$$\frac{dN}{dt} = -\mu_d \cdot N \Leftrightarrow N = N_s \cdot e^{-\mu_d \cdot t}$$

4. YIELDS

$Y_{X/S}$ Substrate to biomass yield (g cells / g substrate)

← Connects the amount of biomass produced and the amount of substrate consumed.

$$Y_{X/S} = -\frac{dX}{dS} \approx -\frac{\Delta X}{\Delta S}$$

$Y_{P/S}$ Substrate to product yield (g product / g substrate)

← Connects the amount of product generated and the amount of substrate consumed.

$$Y_{P/S} = -\frac{dP}{dS} \approx -\frac{\Delta P}{\Delta S}$$

4. YIELDS

Y_{X/O_2} Oxygen to biomass yield (g cells / g oxygen)

← Connects the amount of biomass produced and the amount of oxygen consumed.

$$Y_{X/O_2} = -\frac{dX}{dO_2} \approx -\frac{\Delta X}{\Delta O_2}$$

$Y_{P/X}$ Biomass to product yield (g product / g cells)

← Connects the amount of product generated and the amount of biomass produced.

$$Y_{P/X} = \frac{dP}{dX} \approx \frac{\Delta P}{\Delta X}$$

5. MAINTENANCE COEFFICIENT

DEFINITION:

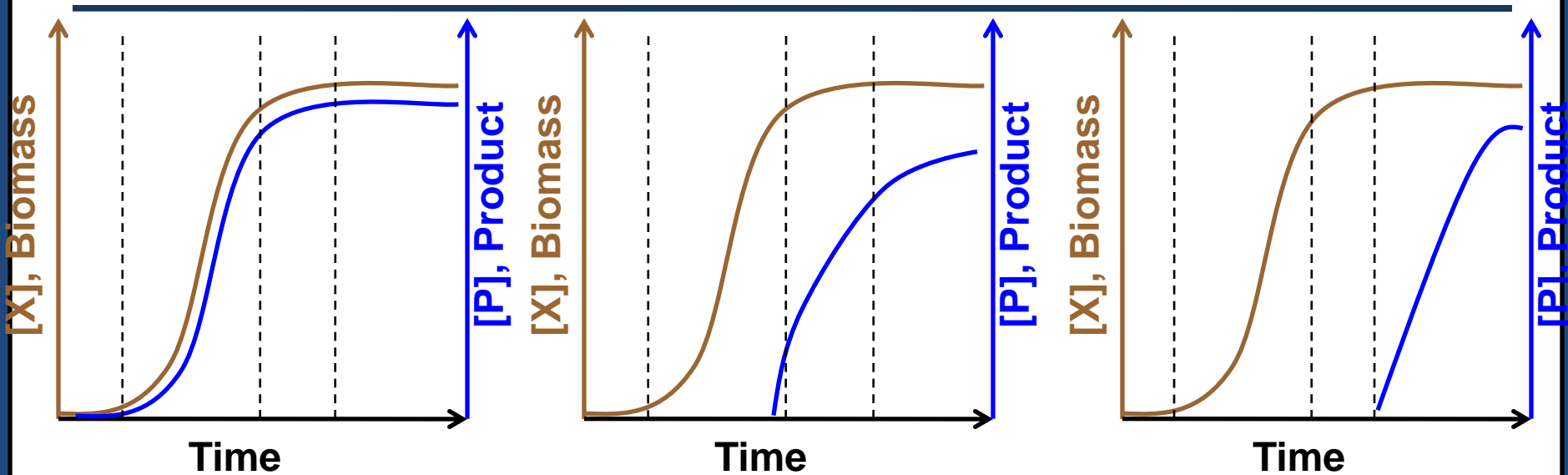
specific rate of substrate consumption for cell maintenance.

$$m = - \frac{\left[\frac{dS}{dt} \right]_{average}}{X}$$

It represents the energy expenditure necessary to:

- Repair cell damage
- Transport of nutrients and products through
- Motility
- Osmolarity control

6. KINDS OF PRODUCTS



**ASSOCIATED
WITH GROWTH**

**PRIMARY
METABOLITE**

**PARTIALLY
ASSOCIATED
WITH GROWTH**

**SECONDARY
METABOLITE**

**NON ASSOCIATED
WITH GROWTH**

6. KINDS OF PRODUCTS

$$q_P = \frac{1}{X} \cdot \frac{dP}{dt}$$

$$\Rightarrow q_P = Y_{P/X} \cdot \mu$$

**ASSOCIATED
WITH GROWTH**

**PRIMARY
METABOLITE**

$$q_P = \alpha \cdot \mu + \beta$$

**PARTIALLY
ASSOCIATED
WITH GROWTH**

**SECONDARY
METABOLITE**

$$q_P = \beta = cte$$

**NON ASSOCIATED
WITH GROWTH**

7. SOLVED OXYGEN

- No oxygen limitation →

Growth rate **doesn't depend** on oxygen concentration

Growth: first-order kinetics

- Oxygen limitations →

Growth **depends on** oxygen concentration

Saturating kinetics

7. SOLVED OXYGEN

- OXYGEN TRANSPORT RATE (OTR)

From gas to the liquid phase:

$$N_{O_2} = k_L a \cdot (C^* - C_L) = OTR$$

- k_L : coefficient of oxygen transfer (cm/h)
- a : interfacial surface between gas and liquid (cm²/cm³)
- $k_L a$: volumetric coefficient of oxygen transfer (h⁻¹)
- C^* : saturation concentration of oxygen (mg/L).
- C_L : concentration of oxygen within the liquid (mg/L).
- N_{O_2} : OTR (mg O₂/(L·h))

7. SOLVED OXYGEN

- OXYGEN UPTAKE RATE(OUR)

By microorganism:

$$OUR = q_{O_2} \cdot X = \frac{\mu_g \cdot X}{Y_{X/O_2}}$$

- q_{O_2} : specific oxygen uptake rate (mg O₂/(g·h))
- Y_{X/O_2} : oxygen to biomass yield (g/g)
- X : biomass concentration (g/L)

7. SOLVED OXYGEN

-MASS BALANCE FOR OXYGEN

Accumulation = Transport – Uptake

$$\frac{dO_2}{dt} = k_L a \cdot (C^* - C_L) - q_{O_2} \cdot X = OTR - OUR$$

1.- GROWTH

2.- CELLS IN BATCH PROCESSES

3.- CELLS IN CONTINUOUS PROCESSES

3.- CELLS IN CONTINUOUS PROCESSES

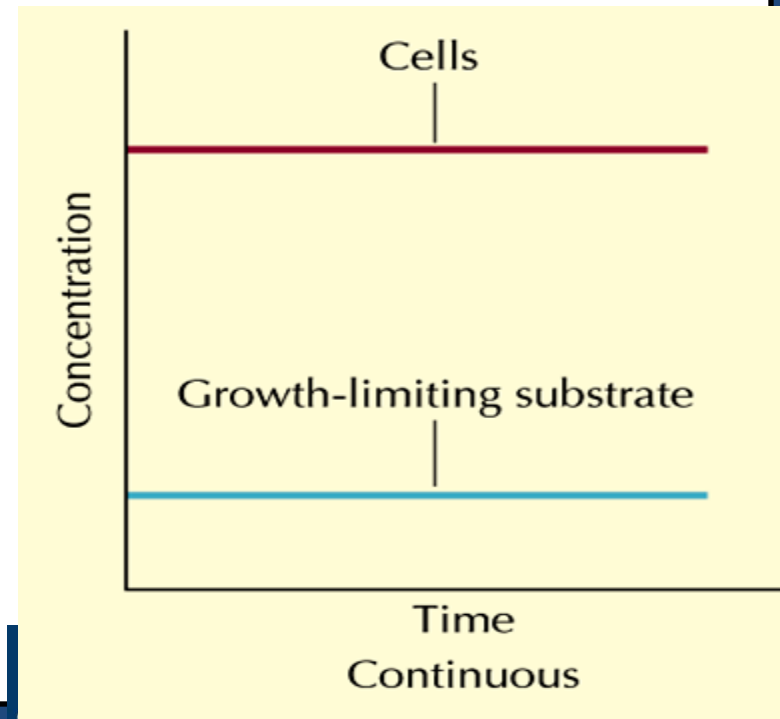
8. MICROBIAL CONTINUOUS REACTORS

- Fresh medium need to be constantly supplied → nutrients
- Biomass and product constantly extracted → avoid inhibition
- Maintaining conditions for long periods

STEADY STATE is reached

$$[S]=[P]= \text{constant}$$

HOMOGENEITY IN PRODUCT QUALITY



8. MICROBIAL CONTINUOUS REACTORS

→ **CULTURE** extended along time

Perfect mixing hypothesis

- **Chemostat**
- **Turbidostat**

Plug flow hypothesis

- **Tubular (PLUG FLOW) reactor**

8. MICROBIAL CONTINUOUS REACTORS

PERFECT MIX HYPOTHESIS:

Easiest approach for Tank Reactor behaviour.

Matter entering the reactor is **instantaneously and homogeneously mixed** so that at each moment the concentration inside the vessel is exactly the same in the outlet current.

No short-circuit, nor dead zones.

8. MICROBIAL CONTINUOUS REACTORS

PERFECT MIX HYPOTHESIS :

Matter entering the reactor is **instantaneously and homogeneously mixed** so that at each moment the concentration inside the vessel is exactly the same in the outlet current.

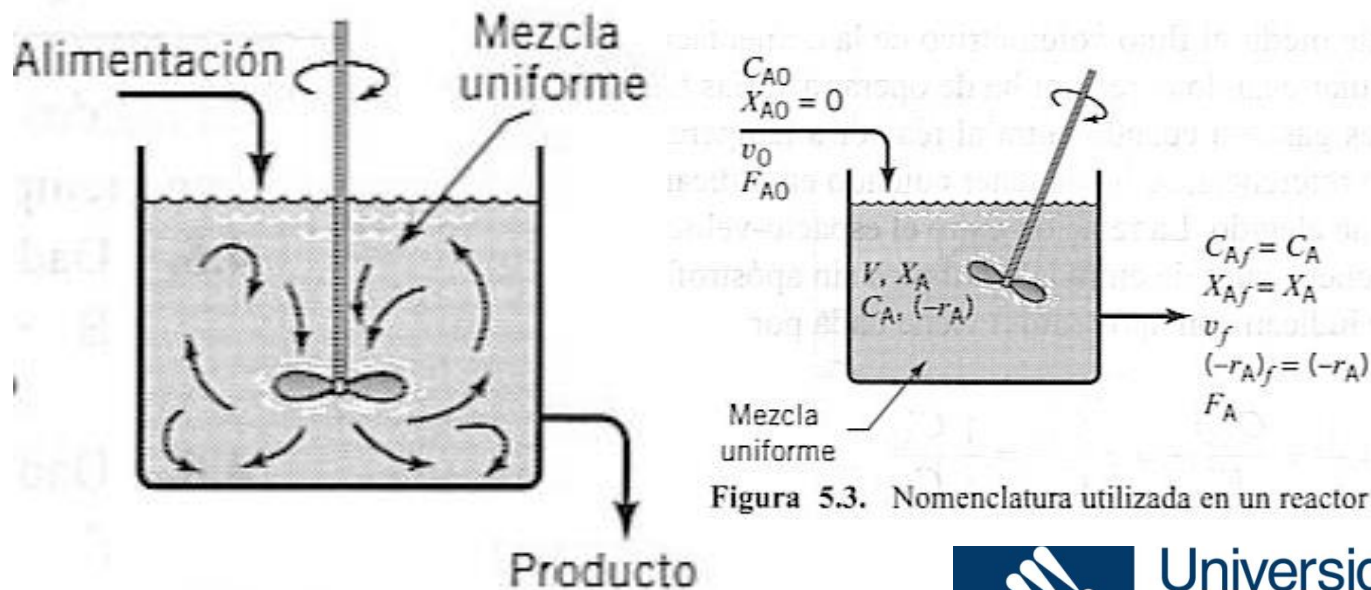


Figura 5.3. Nomenclatura utilizada en un reactor de tanque agitado

8.1 CHEMOSTAT

One limiting nutrient determines growth rate and cell density.

Growth is kept constant by supplying fresh medium with a nutrient at a fixed concentration while extracting culture containing microorganisms with the same rate.

CHEMOSTAT \leftrightarrow CHEMICALLY CONSTANT ENVIRONMENT

8.1 CHEMOSTAT

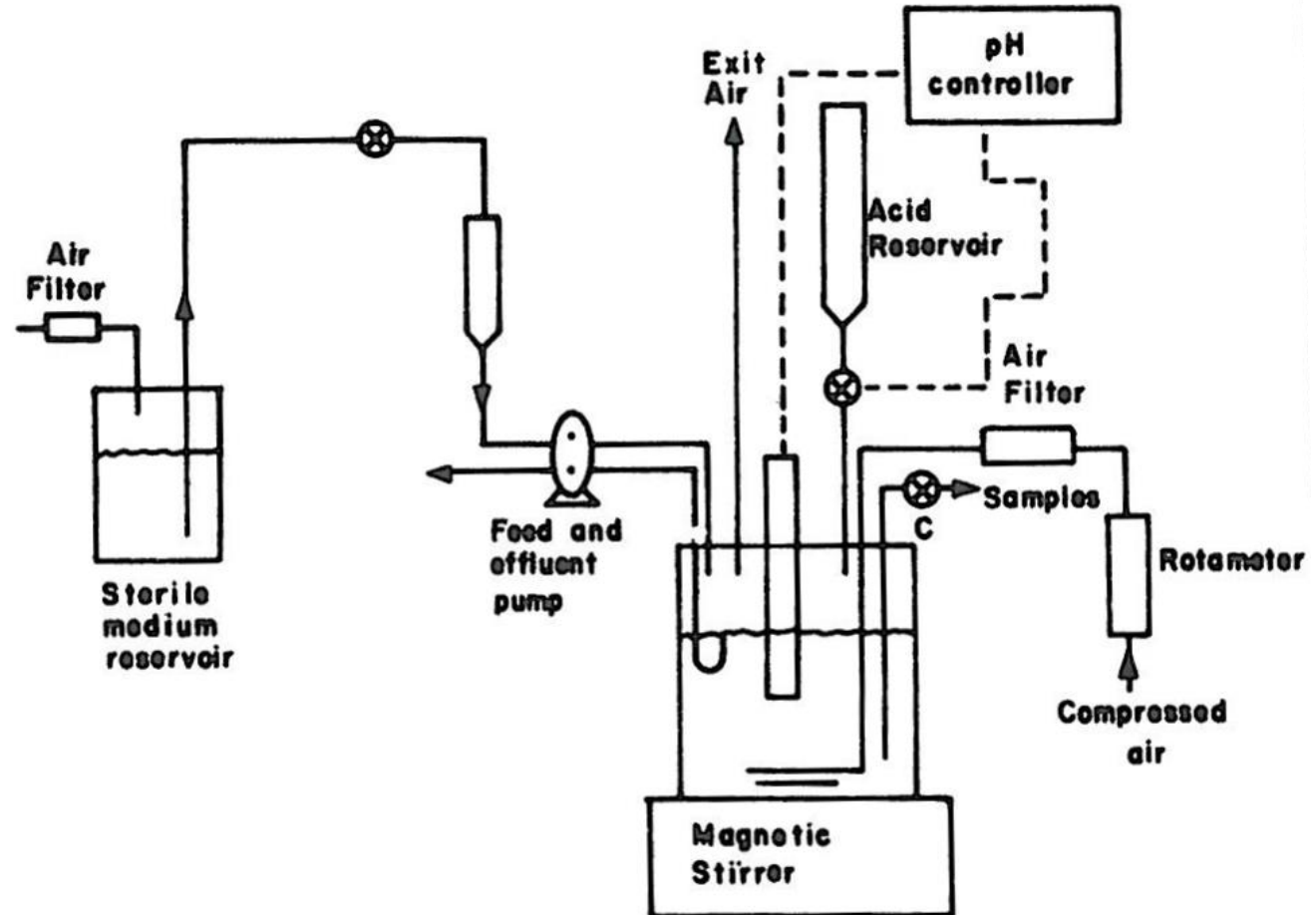


Figure 6.16. A continuous-culture laboratory setup (chemostat). (With permission, from D. I. C. Wang and others, *Fermentation and Enzyme Technology*, John Wiley & Sons, Inc., New York, 1979, p. 99.)

8.2. TURBIDOSTAT

Biomass concentration is maintained constant by measuring the optical density and controlling the inlet current.

In order to avoid changes in reactor volume, the same amounts of culture being removed and medium being added are needed.

TURBIDOSTAT <> DYNAMIC ENVIRONMENT

More difficult control than in chemostat case.

8.2. TURBIDOSTAT

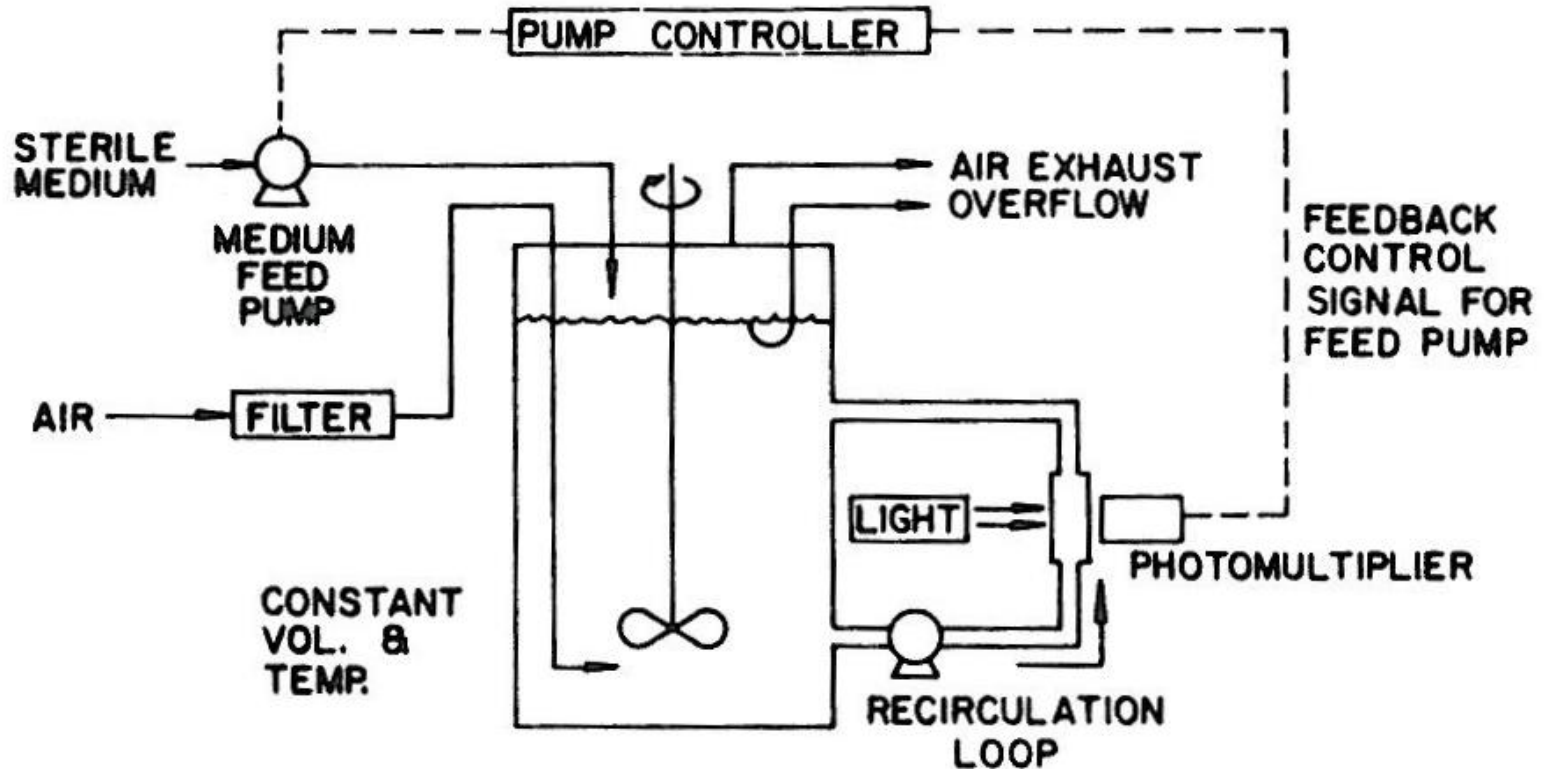


Figure 6.17. Typical laboratory setup for a turbidostat. (With permission, from D. I. C. Wang and others, *Fermentation and Enzyme Technology*, John Wiley & Sons, Inc., New York, 1979, p. 100.)

8.3. PLUG FLOW

PLUG FLOW HYPOTHESIS :

Easiest approach for Tubular Reactor behaviour.

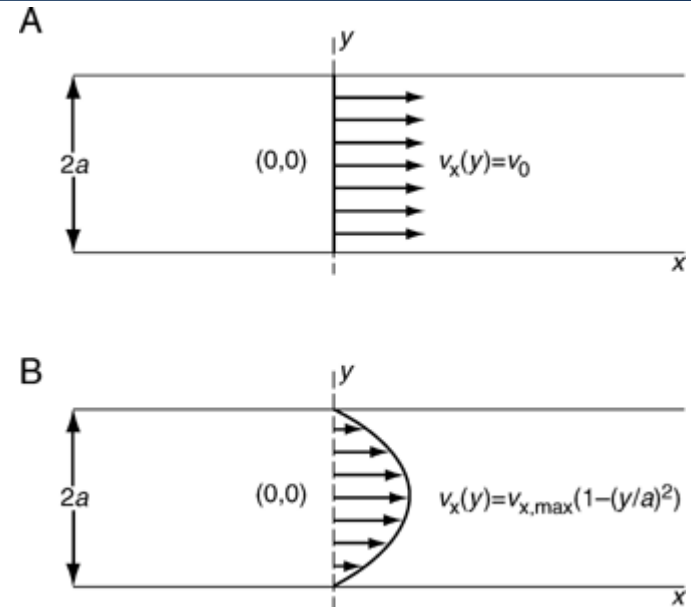
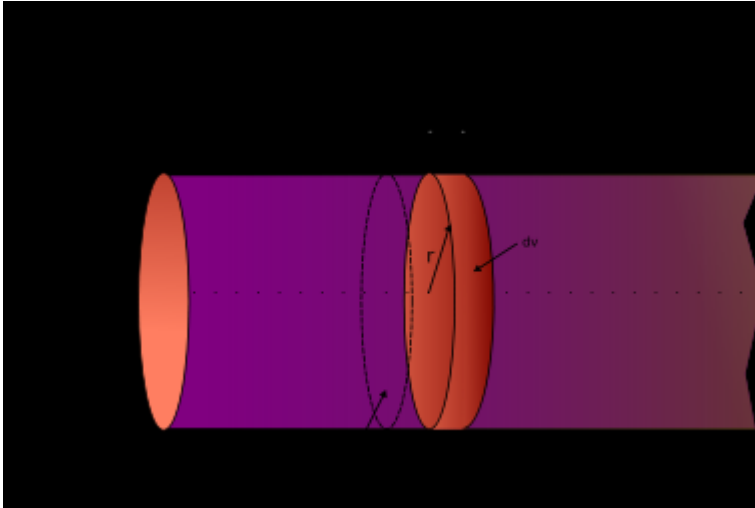
Uniformity along any cross-section in the reactor

→ same speed and fluid properties

(temperature, pressure and composition) .

No axial flow.

8.3 PLUG FLOW



No mixing along this axis between cells inoculated at different times.

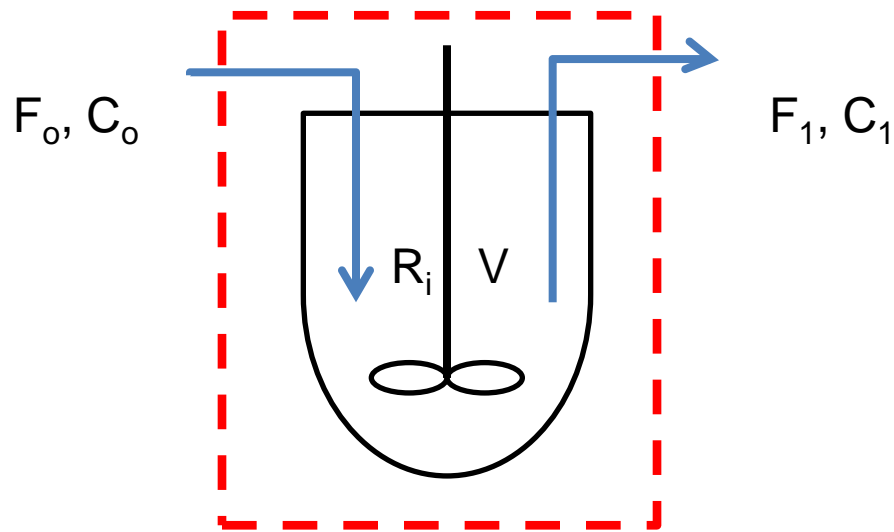
→ Often used in waste treatment.

8.4 IDEAL CHEMOSTAT

CONDITIONS:

- Continuous Flow
- Complete mix
- Stirred tank reactor
- Control of pH, T, ...
- Feeding of a **sterile medium**, without biomass.
- Constant reaction volume

8.4 IDEAL CHEMOSTAT



$$\frac{V \cdot dC_i}{dt} = F_o \cdot C_{i,0} - F_1 \cdot C_{i,1} + V \cdot R_i$$

(i) MASS BALANCE

WITHIN BIOREACTOR for

i component

↓
Accumulation

↓
Inputs

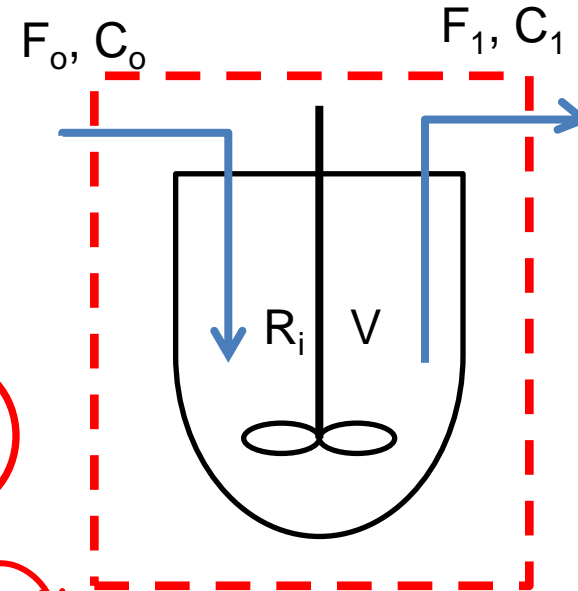
↓
Outputs

↓
Generation

8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR *para biomasa*

$$\frac{V \cdot dC_i}{dt} = F_o \cdot C_{i,0} - F_1 \cdot C_{i,1} + V \cdot R_i$$

$$\frac{V \cdot d[X]}{dt} = F_o \cdot [X]_0 - F_1 \cdot [X]_1 + V \cdot R_X$$


In order to calculate R_x , which are reactions where biomass is involved?



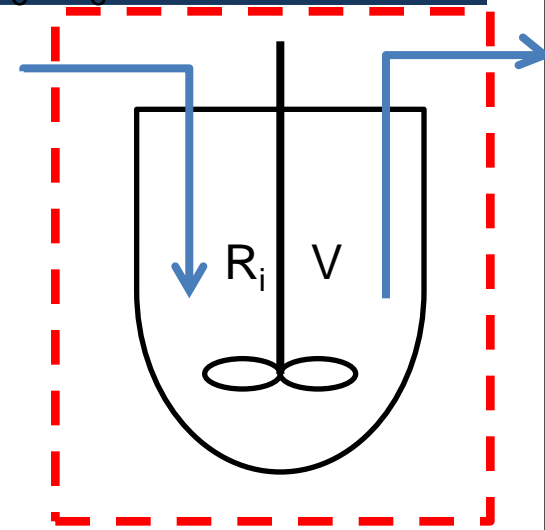
8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR for biomass:

$$\frac{V \cdot dC_i}{dt} = F_o \cdot C_{i,0} - F_1 \cdot C_{i,1} + V \cdot R_i$$

$$\frac{V \cdot d[X]}{dt} = F_o \cdot [X]_0 - F_1 \cdot [X] + V \cdot R_X$$

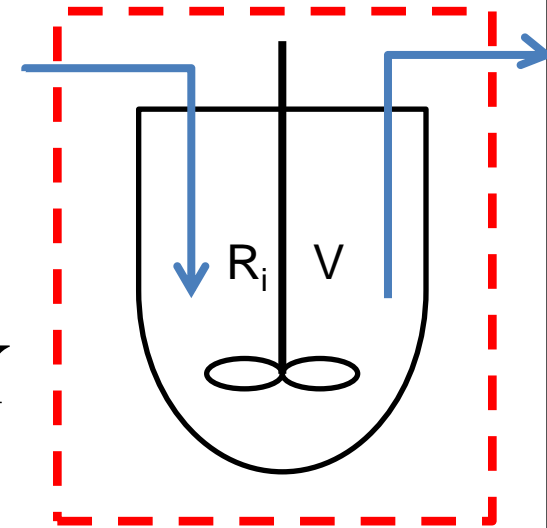
$$\frac{V \cdot d[X]}{dt} = F_o \cdot [X]_0 - F_1 \cdot [X] + V \cdot \mu_{net} \cdot X$$



8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR for biomass:

$$\frac{V \cdot d[X]}{dt} = F_o \cdot [X]_0 - F_1 \cdot [X] + V \cdot \mu_{net} \cdot X$$



- $[X]_0$: biomass concentration within Input currents (DCW g/L)
- F_o : Input flow (L/h)
- $[X]$: biomass concentration within output currents (DCW g/L)
- F : Output flow (L/h)

8.4 IDEAL CHEMOSTAT

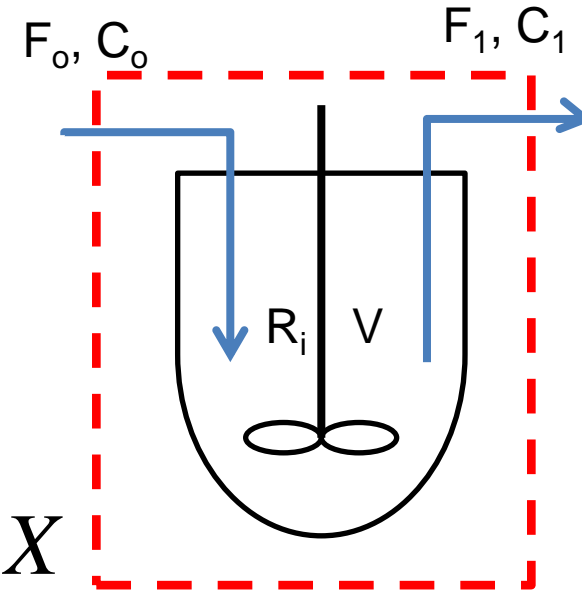
(i) MASS BALANCE WITHIN BIOREACTOR for biomass:

- $F_0 = F_1 = F \rightarrow V = \text{constant}$

$$\frac{V \cdot d[X]}{dt} = F_0 \cdot [X]_0 - F_1 \cdot [X] + V \cdot \mu_{net} \cdot X$$

⇓

$$\frac{V \cdot d[X]}{dt} = F \cdot [X]_0 - F \cdot [X] + V \cdot \mu_{net} \cdot X$$



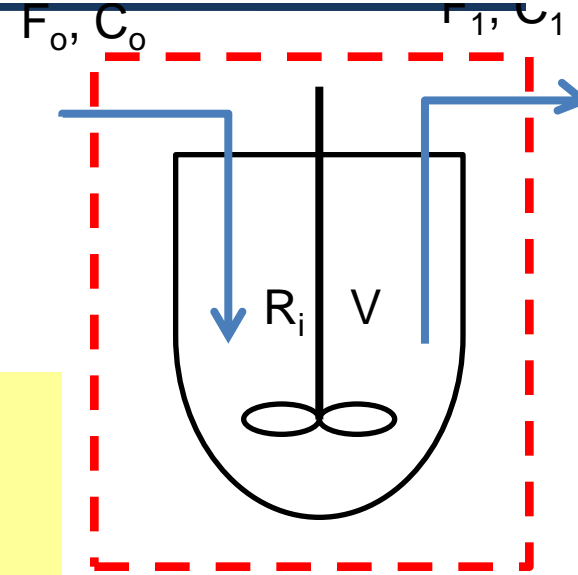
8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR **for biomass:**

DILUTION RATE:

Quotien between Fed flow and volumen of reactor:

$$D = F/V$$



$$\frac{d[X]}{dt} = D \cdot [X]_0 - D \cdot [X] + \mu_{net} \cdot X$$

$$\frac{d[X]}{dt} = D \cdot [X]_0 + (\mu_{net} - D) \cdot X$$

8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR **for biomass:**

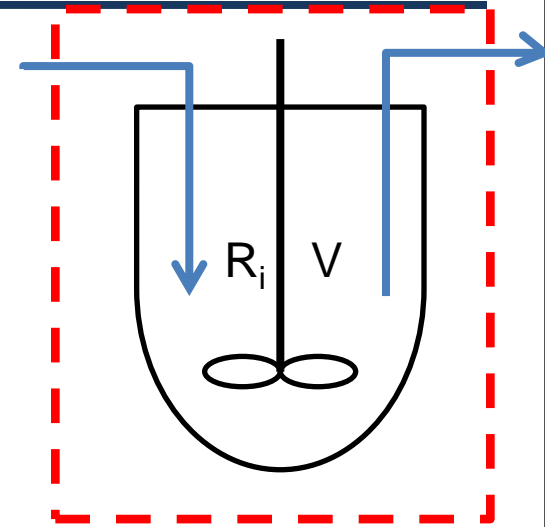
HYPOTHESIS:

- Sterile feeding $\rightarrow [X]_0 = 0$ (dw g/L)

$$\frac{d[X]}{dt} = D \cdot [X]_0 + (\mu_{net} - D) \cdot X$$

- Insignificant endogenous metabolism $\rightarrow \mu_d = 0$

$$\frac{d[X]}{dt} = (\mu_g - D) \cdot X$$



8.4 IDEAL CHEMOSTAT

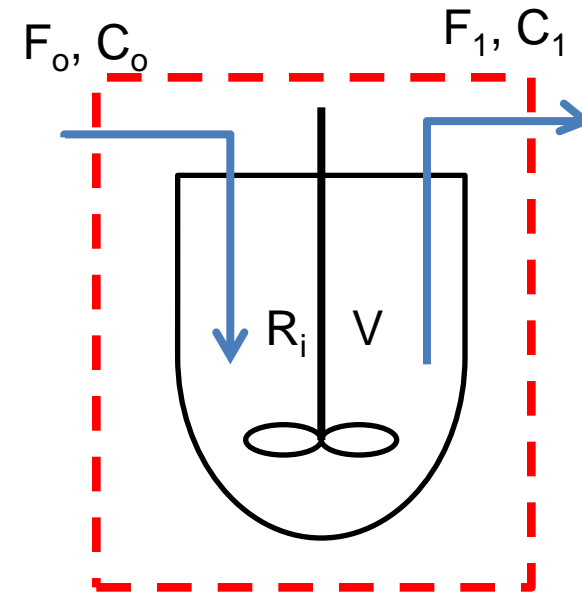
(i) MASS BALANCE WITHIN BIOREACTOR for biomass:

HYPOTHESIS:

- Steady State $\rightarrow d[X]/dt = 0$

$$\frac{d[X]}{dt} = (\mu_g - D) \cdot X$$

$$0 = (\mu_g - D) \cdot X \Rightarrow D = \mu_g$$

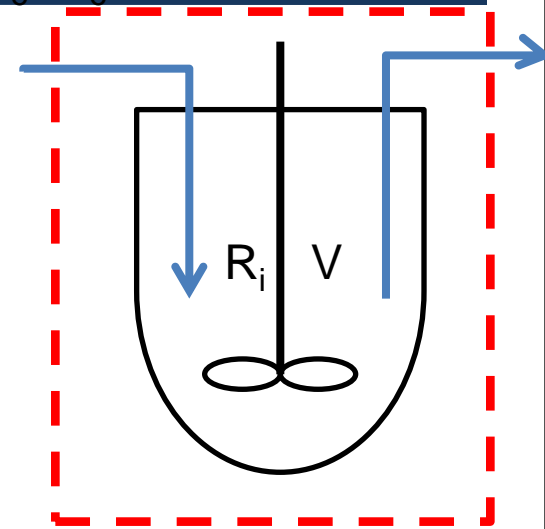


8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR **for biomass:**

HYPOTHESIS:

- Sterile feeding $\rightarrow [X]_0 = 0$ (dw g/L)
- Insignificant endogenous metabolism $\rightarrow \mu_d = 0$
- Steady State $\rightarrow d[X]/dt = 0$



$$D = \mu_g = \frac{\mu_m \cdot [S]}{K_s + [S]}$$

Monod equation:

Describes growth kinetics when there is a

limiting nutrient, S .

8.4 IDEAL CHEMOSTAT

(i) MASS BALANCE WITHIN BIOREACTOR **for biomass:**

$$D = \frac{\mu_m \cdot [S]}{K_s + [S]} \Rightarrow [S] = \frac{K_s \cdot D}{\mu_m - D}$$

$$D < \mu_m$$

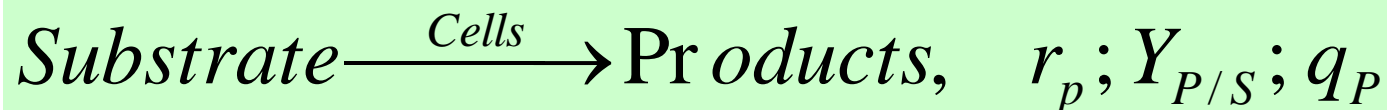
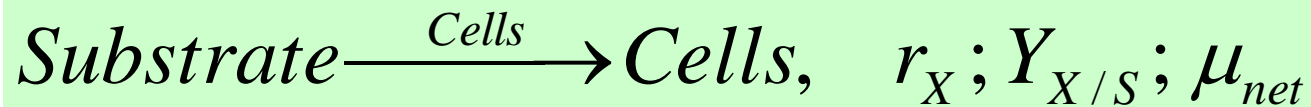
8.4 IDEAL CHEMOSTAT

(ii) MASS BALANCE WITHIN BIOREACTOR for substrate

$$\frac{V \cdot dC_i}{dt} = F \cdot C_i - F \cdot C_i + V \cdot R_i$$

$$\frac{V \cdot d[S]}{dt} = F \cdot [S]_0 - F \cdot [S] + V \cdot R_S$$

In order to calculate R_S , which reaction need to be considered?



8.4 IDEAL CHEMOSTAT

(ii) MASS BALANCE WITHIN BIOREACTOR **for substrate:**

$$\frac{V \cdot d[S]}{dt} = F \cdot [S]_0 - F \cdot [S] + V \cdot R_S$$



$$R_S = -\mu_g \cdot X \cdot \frac{1}{Y_{X/S}^{\max}} - q_P \cdot X \cdot \frac{1}{Y_{P/S}}$$



(ii) MASS BALANCE WITHIN BIOREACTOR **for substrate:**

$$\frac{V \cdot d[S]}{dt} = F \cdot [S]_0 - F \cdot [S] - V \cdot \mu_g \cdot X \cdot \frac{1}{Y_{X/S}^{\max}} - V \cdot q_P \cdot X \cdot \frac{1}{Y_{P/S}}$$

- If extracellular generation of products is negligible $\rightarrow q_P = 0$.
- Steady state $\rightarrow d[S]/dt = 0$

$$0 = F \cdot [S]_0 - F \cdot [S] + V \cdot \mu_g \cdot X \cdot \frac{1}{Y_{X/S}^{\max}}$$

$$0 = D \cdot [S]_0 - D \cdot [S] - \mu_g \cdot X \cdot \frac{1}{Y_{X/S}^{\max}} \Rightarrow D \cdot ([S]_0 - [S]) = \frac{\mu_g \cdot X}{Y_{X/S}^{\max}}$$



8.4 IDEAL CHEMOSTAT

(ii) MASS BALANCE WITHIN BIOREACTOR **for substrate:**

$$D \cdot ([S]_0 - [S]) = \frac{\mu_g \cdot X}{Y_{X/S}^{\max}}$$

- No Endogenous metabolism and steady state $\rightarrow D = \mu_g$

$$X = Y_{X/S}^{\max} \cdot ([S]_0 - [S])$$

$$X = Y_{X/S}^{\max} \cdot \left([S]_0 - \frac{K_s \cdot D}{\mu_m - D} \right)$$



A photograph of an industrial facility featuring several large, cylindrical, light-colored storage tanks. The tanks are arranged in a row, with the closest one being the most prominent. A metal staircase and various pipes are visible on the right side of the tanks. In the background, there are other industrial buildings and a clear blue sky with scattered white clouds. A dark blue banner with white text is overlaid on the middle of the image.

ANY QUESTION?

KINETICS AND METABOLIC STOICHIOMETRY



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SECTION II: KINETICS AND BIOREACTOR DESIGN:
LESSON 9.2. - Enzymatic kinetics, microbial kinetics and metabolic stoichiometry – Alive cells in bioprocesses



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